**Abstract**

This paper presents a method of estimating political actors’ positions in a latent social space from dynamic network data. The proposed latent path model is a natural extension of the latent space model for static networks [2], allowing political actors’ locations in the social space to shift over time to reflect the changes in the structure of ties in the network. Unlike alternative models of [3,4], the proposed model does require a Markov assumption and a lag structure need not be specified. The proposed model is validated through Monte Carlo simulations and applied to Fowler’s Senate cosponsorship data (reported below) and Polish party switching (forthcoming).

**Latent Space Model**

- **Intuition**: Closer actors are more likely to have more intensive relations. For example, in the graph below, the models estimates positions for B and C that are closer to each other than they are to A because of the stronger relation between B and C.

- **Formal representation**: If i and j are actors in graph Y:
  \[ \Pr(y_{ij} | x_i, x_j, z_i, z_j) = f(P_i, x_{ij}, d(x_i, z_i)) \]

  where \( y_{ij} \) indicates the strength of relation between i and j; \( x_{ij} \) are dyadic covariates; \( \beta \) is a vector of coefficients on those covariates; and \( d(z_i, z_j) \) is a function measuring the distance between i and j in the latent social space, and it’s assumed that \( \sigma^2 < 0 \).

**Latent Path Model**

- **Intuition**: Rather than estimating a single point for each realization of the network, estimate a path through the latent social space for each actor.

Here the top panel shows the results of estimating three separate latent space models for each realization of the network. The bottom panel shows the estimation of a latent path model. The model is less likely to overfit latent positions and statistically more efficient than estimating separate latent space models.

**Latent Path Model**

- **Formal representation**: Latent path model extends the latent space model by incorporating a time component:
  \[ \Pr(y_{ijt} | x_{ijt}, z_i, z_j) = f(P_i, x_{ijt}, d(x_i, z_i)) \]

Where \( z_i \) is now a function representing a path, or trajectory, in the latent social space: \( z_i(t) = g(z_i(t-1)) \). For example, a linear trajectory may be defined as \( z_i(t) = a z_i(t-1) + b z_{j(t-1)} + d_x(t) \), where \( s_i(t) \) is a vector indicating the amount and direction of movement of the actor in the social space between network realizations. More complicated, non-linear models are possible.

- **Joint likelihood**: The model fits naturally into the GLM framework and can be estimated with standard maximum likelihood. For example, let \( \mu = \beta x_{ijt} - d_i z_i, z_j \) then the log-likelihood for a Poisson model would be
  \[ \log L(\cdot) = \sum_{i,j} \left\{ -\exp(\mu + y_{ijt} - \log(y_{ijt}) \right\} \]

- **Identification**: Distances are invariant to scale, translation, and rotation. Identification requires fixing the paths of k(k+1) actors.

**Monte Carlo Simulations**

- **Fixed network characteristics**: - Undirected affiliation networks - Valued edges of shared affiliations - Expected edge value, \( \beta = 2 \) - 2-dimensional social space - Poisson, linear trajectory latent path model

- **Varying network characteristics**: - Number of actors: \( n \in \{25, 50, 100\} \) - Number of time periods: \( T \in \{2, 5, 10\} \) - 1000 Simulations - Starting positions: \( z_i \sim N(0, 1) \) - Trajectory vector: \( k \sim N(0, 0.25^2) \)

**Monte Carlo Results**

**Application: Senate Cosponsorships**

- **Data**: Fowler Senate Cosponsorship data [1]. - 93rd-108th Congresses (1973-2004); 298 Senators; 52,104 bills and nearly 386,000 cosponsorships. - Affiliation network transformed into a discrete-valued, one-mode networks of shared cosponsorships.

- **Model**: - Poisson model - 1-dimensional latent path model with a linear trajectory.

**Application: Senate Cosponsorships**

**Further Research**

- **Data**: Complete analysis of Polish party switching.
- **Monte Carlo**: Extend to logistic model; include 1-dimensional models; develop non-linear trajectories.
- **Estimation**: Explore more efficient methods for estimation of high-dimensional problems; identification through MDS rather than fixing locations of particular actors.
- **Reporting**: Improve graphical representation of results.

**References**